

Physical Interpretation of Laser-Induced Suppression of Quantum Tunneling

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We revisit the problem of laser-induced suppression of quantum dynamical tunneling in a model system studied by Kilin *et al.* [Phys. Rev. Lett. 76 (1996) 3297]. This quantum system consists of a ground state symmetric double-well potential which is coupled by a strong laser field to an excited state asymmetric double-well potential. By analyzing the assumptions used in their analysis we show that the suppression of quantum dynamical tunneling can be explained with the use of dark and bright states of the system. We also generalize the system and the conditions for suppression of quantum tunneling and show that, in certain cases, suppression can occur regardless the characteristics of the excited potential surface.

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One of the most interesting features of a Λ -type quantum system (a system with two lower states and one upper state) driven by two laser fields is the appearance of a dark eigenstate of the system under conditions of two-photon resonance [1,2]. The dark state is a linear combination of only the two lower states of the system, and not the excited state. Dark states also exist in systems where interfering dissipative processes are present [3]. Numerous interesting phenomena are associated with dark states, such as coherent population trapping [4] and transfer [5,6], electromagnetically induced transparency [7,8] and propagation of matched pulses [9], propagation of soliton-like pulses in multi-level media (like “simultons” [10] and “adiabatons” [11,12]), intrinsic transparency [13], lasing without inversion [14], creation of radiation fields with matched photon statistics [15,16], quenching of spontaneous emission [17–19] or resonance fluorescence [20,21], velocity selective laser cooling [22] and others.

In a recent article, Kilin *et al.* [23] showed that it is possible to manipulate coherently the process of quantum dynamical tunneling [24] in a model system consisting of a ground symmetric double-well potential $V_g(x)$ and an excited asymmetric double-well potential $V_e(x)$ (see Fig. 1). It is well known (see, for example, ref. [25]) that if the wavepacket is initially localized in one of the two ground state potential wells, then it will tunnel through the barrier to the other well and will ultimately oscillate between the two potential wells. If now the ground state well is coupled to the excited state well by a strong laser field, this will lead to the localization of the wavepacket in one of the wells [23], leading to suppression of quantum dynamical tunneling. In this article, we show that the phenomenon of suppression of quantum tunneling which was discussed by Kilin *et al.* [23] has its origin in the properties of the Λ -type system. This is shown by analyzing the assumptions used in the above article and transforming the problem to a basis of dark and bright states of the system [1]. We also generalize the system and show that one assumption is required for the suppression of quantum tunneling, if the properties of the dark state is properly exploited.

We begin with an analysis of the process of quantum dynamical tunneling in our model system [24,25]. We consider only the ground state symmetric double-well potential shown in Fig. 1 and recall that the wave functions of states $|1\rangle$ and $|2\rangle$ can be written as symmetric and anti-symmetric superpositions of the localized wave functions in each of the potential wells,

$$\psi_1(x) = \frac{1}{\sqrt{2}} [\phi(x) + \phi(-x)] , \quad (1)$$

$$\psi_2(x) = \frac{1}{\sqrt{2}} [\phi(x) - \phi(-x)] . \quad (2)$$

Here, $\psi_1(x) = \langle x|1\rangle$, $\psi_2(x) = \langle x|2\rangle$ and $\phi(x)$ ($\phi(-x)$) is the wave function in the right- (left-) side of the potential well. At $t = 0$ we suppose that the wave function of the system is written as a superposition of the two states $|1\rangle$ and $|2\rangle$, $|\psi(t=0)\rangle = c_1|1\rangle + c_2|2\rangle$, leading to

$$\psi(x, t=0) = \frac{c_1 + c_2}{\sqrt{2}} \phi(x) + \frac{c_1 - c_2}{\sqrt{2}} \phi(-x) , \quad (3)$$

with $\psi(x, t) = \langle x|\psi(t)\rangle$. The evolution of the system at any time t is given by (we use units such that $\hbar = 1$)

$$\begin{aligned} \psi(x, t) &= c_1 \psi_1(x) e^{-i\omega_1 t} + c_2 \psi_2(x) e^{-i\omega_2 t} \\ &= \frac{e^{-i(\omega_1 + \omega_2)t/2}}{\sqrt{2}} \left[\left(c_1 e^{i\delta t/2} + c_2 e^{-i\delta t/2} \right) \phi(x) \right. \\ &\quad \left. + \left(c_1 e^{i\delta t/2} - c_2 e^{-i\delta t/2} \right) \phi(-x) \right] , \end{aligned} \quad (4)$$

where ω_i , ($i = 1, 2$) is the energy of state $|i\rangle$ and $\delta = \omega_2 - \omega_1$ is the separation energy of states $|2\rangle$ and $|1\rangle$. This energy separation depends on the width of the potential barrier between the wells. Therefore, if the wavepacket is initially localized on the left [$\psi(x, t=0) = \phi(-x)$], ($c_1 = -c_2 = 1/\sqrt{2}$) then

$$\begin{aligned} \psi(x, t) &= e^{-i(\omega_1 + \omega_2)t/2} \left[i \sin(\delta t/2) \phi(x) \right. \\ &\quad \left. + \cos(\delta t/2) \phi(-x) \right] , \end{aligned} \quad (5)$$

so that

$$P_L(t) = \cos^2(\delta t/2) \quad , \quad P_R(t) = \sin^2(\delta t/2) \quad , \quad (6)$$

with $P_L(t)$ ($P_R(t)$) being the probability for the wavepacket to be localized on the left- (right-) side of the well. If now the wavepacket is initially localized on the right [$\psi(x, t=0) = \phi(x)$], ($c_1 = c_2 = 1/\sqrt{2}$) then

$$\begin{aligned} \psi(x, t) = e^{-i(\omega_1 + \omega_2)t/2} & \left[\cos(\delta t/2) \phi(x) \right. \\ & \left. + i \sin(\delta t/2) \phi(-x) \right] \quad , \end{aligned} \quad (7)$$

and

$$P_L(t) = \sin^2(\delta t/2) \quad , \quad P_R(t) = \cos^2(\delta t/2) \quad . \quad (8)$$

So, in both of the above cases the wavepacket will oscillate between the two potential wells and will be localized at each of the wells only at certain times $t = n\pi/\delta$, with n being an integer.

We now suppose that a coherent, step pulse laser field couples states $|1\rangle$ and $|2\rangle$ with a state $|3\rangle$ belonging to another potential well surface (see Fig. 1). The Hamiltonian of this system, in the the rotating wave approximation, is given by

$$\begin{aligned} H = \sum_{i=1}^3 \omega_i |i\rangle\langle i| + & \left[\Omega_1 e^{i\omega t} |1\rangle\langle 3| \right. \\ & \left. + \Omega_2 e^{i\omega t} |2\rangle\langle 3| + \text{H.c.} \right] - i\frac{\gamma}{2} |3\rangle\langle 3| \quad . \end{aligned} \quad (9)$$

Here, $\Omega_i = -\mu E \int dx \psi_i^*(x) \psi_3(x)$, ($i = 1, 2$) is the Rabi frequency of the $|i\rangle \rightarrow |3\rangle$ transition. The Rabi frequency has been obtained using the adiabatic approximation and is assumed to be real. Also, μ is the electric dipole matrix element, E is the electric field amplitude and ω is the angular frequency of the laser field. Finally, γ denotes the decay of the excited state, which is assumed to occur outside of the system and has been added phenomenologically to the Hamiltonian. The wave function of the system is expanded in terms of the “bare” state vectors as $|\psi(t)\rangle = \sum_{i=1}^3 a_i(t) |i\rangle$. We substitute the above Hamiltonian and wave function into the time-dependent Schrödinger equation and after a transformation we obtain

$$i\dot{\mathbf{b}}(t) = \mathbf{H}\mathbf{b}(t) \quad , \quad (10)$$

where $\mathbf{b}(t) = (b_1(t), b_2(t), b_3(t))^T$ and

$$\mathbf{H} = \begin{pmatrix} \delta_1 & 0 & \Omega_1 \\ 0 & \delta_2 & \Omega_2 \\ \Omega_1 & \Omega_2 & -\frac{i}{2}\gamma \end{pmatrix} \quad , \quad (11)$$

with $\delta_i = \omega - \omega_3 + \omega_i$, ($i = 1, 2$) being the detuning of the $|i\rangle \leftrightarrow |3\rangle$ transition and $a_1(t) = b_1(t)e^{-i(\omega_3 - \omega)t}$,

$a_2(t) = b_2(t)e^{-i(\omega_3 - \omega)t}$, $a_3(t) = b_3(t)e^{-i\omega_3 t}$. We also note that $\delta_2 = \delta_1 + \delta$. The Hamiltonian \mathbf{H} is the same as that used by Kilin *et al.* [23] if $\gamma = 0$ (see Eq. (10) of ref. [23]). It is also the Hamiltonian of a Λ -type atomic system driven by a single laser field [1,2,4].

We now define the “dark” $|-\rangle$ and “bright” $|+\rangle$ states as

$$|+\rangle = \frac{1}{\Omega} (\Omega_1 |1\rangle + \Omega_2 |2\rangle) \quad , \quad (12)$$

$$|-\rangle = \frac{1}{\Omega} (\Omega_2 |1\rangle - \Omega_1 |2\rangle) \quad , \quad (13)$$

with $\Omega = \sqrt{\Omega_1^2 + \Omega_2^2}$. In this basis the equations for the probability amplitudes are written as

$$\begin{aligned} i\dot{b}_+(t) = & \frac{\delta_1 \Omega_1^2 + \delta_2 \Omega_2^2}{\Omega^2} b_+(t) + \frac{(\delta_1 - \delta_2) \Omega_1 \Omega_2}{\Omega^2} b_-(t) \\ & + \Omega b_3(t) \quad , \end{aligned} \quad (14)$$

$$i\dot{b}_-(t) = \frac{(\delta_1 - \delta_2) \Omega_1 \Omega_2}{\Omega^2} b_+(t) + \frac{\delta_1 \Omega_1^2 + \delta_2 \Omega_2^2}{\Omega^2} b_-(t) \quad , \quad (15)$$

$$i\dot{b}_3(t) = -i\frac{\gamma}{2} b_3(t) + \Omega b_+(t) \quad . \quad (16)$$

with

$$b_+(t) = \frac{\Omega_1}{\Omega} b_1(t) + \frac{\Omega_2}{\Omega} b_2(t) \quad , \quad (17)$$

$$b_-(t) = \frac{\Omega_2}{\Omega} b_1(t) - \frac{\Omega_1}{\Omega} b_2(t) \quad . \quad (18)$$

Then, the wavefunction of the system can be written as

$$\begin{aligned} |\psi(t)\rangle = & [b_+(t)|+\rangle + b_-(t)|-\rangle] e^{-i(\omega_3 - \omega)t} \\ & + b_3(t) e^{-i\omega_3 t} |3\rangle \quad . \end{aligned} \quad (19)$$

We immediately note that if $\delta_2 \approx \delta_1$ state $|+\rangle$ couples only to state $|3\rangle$ and state $|-\rangle$ does not couple to either states $|+\rangle$ or $|3\rangle$. Therefore, if $\gamma \neq 0$ states $|+\rangle$ and $|3\rangle$ decay but the dark state $|-\rangle$ does not. Condition $\delta_2 \approx \delta_1$ is the well-known dark state condition of the Λ -type system [1,2,4]. Only when this condition is satisfied does the Hamiltonian \mathbf{H} have a real, stable eigenvalue and coherent population trapping is possible.

Kilin *et al.* [23] assumed that $\delta_2 \approx \delta_1$, or in other words that $\delta_1 + \delta \approx \delta_1$. (Note that after Eq. (10) in ref. [23], δ does not appear in the equations.) In addition, they assumed that the excited state belongs to an asymmetric quantum-well potential so that its wave function is localized on the left such that

$$\begin{aligned} \Omega_1 = & -\mu E \int dx \psi_1^*(x) \psi_3(x) \\ \approx & -\frac{\mu E}{\sqrt{2}} \int dx \phi^*(-x) \psi_3(x) \quad , \end{aligned} \quad (20)$$

$$\begin{aligned} \Omega_2 = & -\mu E \int dx \psi_2^*(x) \psi_3(x) \\ \approx & \frac{\mu E}{\sqrt{2}} \int dx \phi^*(-x) \psi_3(x) = -\Omega_1 \quad . \end{aligned} \quad (21)$$

Finally, they considered only the case of a metastable excited state, i.e. that $\gamma = 0$. With these assumptions $|+\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$, $\psi_+(x) = \langle x|+\rangle = \phi(-x)$ and $|-\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$, $\psi_-(x) = \langle x|-\rangle = \phi(x)$. Hence, if the wavepacket is initially localized on the left it will remain localized on the left and will simply oscillate between states $|+\rangle$ and $|3\rangle$. If now the wavepacket is initially localized on the right, it will remain on the right as state $|-\rangle$ is uncoupled from states $|+\rangle$ and $|3\rangle$. Therefore, suppression of quantum dynamical tunneling occurs.

How important are the above approximations? If the upper state is not metastable ($\gamma \neq 0$) and the system is initially localized on the left it will simply leave the system as the upper state decays. However, if the system is initially localized on the right it will remain there, because the dark state $|-\rangle$ does not couple to the decaying state $|3\rangle$. If now the system starts from an arbitrary superposition of $|1\rangle$ and $|2\rangle$, only the part of the superposition related to state $|-\rangle$ will be localized on the right, i.e.

$$P_L(t \rightarrow \infty) = 0, \quad (22)$$

$$P_R(t \rightarrow \infty) = P_R(t) = \frac{1}{2} [1 + 2\text{Re}(c_1 c_2^*)]. \quad (23)$$

So, localization can occur even if the upper state decays out of the system.

When the excited state does not belong to an asymmetric quantum well leading to $|\Omega_1| \neq |\Omega_2|$ and the system is initially prepared in the dark state $|-\rangle$ then a part of the wavepacket will remain localized on the left and another part will remain localized on the right of the lower potential well. In addition, no decay will occur as in this case state $|-\rangle$ does not couple to states $|+\rangle$ and $|3\rangle$. The localization probabilities for the system initially in the dark state are given by

$$P_L(t) = \frac{(\Omega_1 + \Omega_2)^2}{2\Omega^2}, \quad (24)$$

$$P_R(t) = \frac{(\Omega_1 - \Omega_2)^2}{2\Omega^2}. \quad (25)$$

For any other initial condition, the overlap of the initial state with the dark state $|-\rangle$ will determine the degree of localization. In this case decay will occur out of the system and the localization probabilities will be given by

$$P_L(t \rightarrow \infty) = |\Omega_2 c_1 - \Omega_1 c_2|^2 \frac{(\Omega_1 + \Omega_2)^2}{2\Omega^4}, \quad (26)$$

$$P_R(t \rightarrow \infty) = |\Omega_2 c_1 - \Omega_1 c_2|^2 \frac{(\Omega_1 - \Omega_2)^2}{2\Omega^4}. \quad (27)$$

For example, if the wavepacket is initially localized on the left then after a transient period where damped tunneling oscillations will occur the system will be localized on both left- and right-side well potential with probabilities

$$P_L(t \rightarrow \infty) = \frac{(\Omega_1 + \Omega_2)^4}{4\Omega^4}, \quad (28)$$

$$P_R(t \rightarrow \infty) = \frac{(\Omega_1^2 - \Omega_2^2)^2}{4\Omega^4}. \quad (29)$$

Therefore, regardless of the shape of the upper potential well, localization of the wavepacket is possible. Even for the case that the upper state is not bound but is a continuum, suppression of quantum tunneling could occur. In that case the problem reduces to that of laser-induced continuum structure [3] and dark states that lead to wavepacket localization can occur in this case, too.

Obviously only the condition $\delta_2 \approx \delta_1$ is crucial here because only under this assumption a dark state is formed. This condition can be fulfilled, for example in molecular systems where $\delta \approx 10^{-5} - 10^{-4}$ eV and δ_1 or δ_2 could be a few orders of magnitude larger for optical transitions.

In conclusion, we have re-examined the prototype system of Kilin *et al.* [23] for laser-induced suppression of quantum tunneling. Using an analysis in dark and bright states we have shown that there is only one important condition for suppression of tunneling and localization of the wavepacket. This condition leads to a dark state in the system. Hence, the phenomenon of laser-induced suppression of quantum dynamical tunneling should also be added to the list of the phenomena that occur via the creation of dark states. Before closing we note that quantum tunneling oscillations, similar to that studied in this article, have been predicted in Bose-Einstein condensates of atomic gases using double-well trapping potentials [26]. It may be therefore possible to use the idea presented here to suppress these oscillations.

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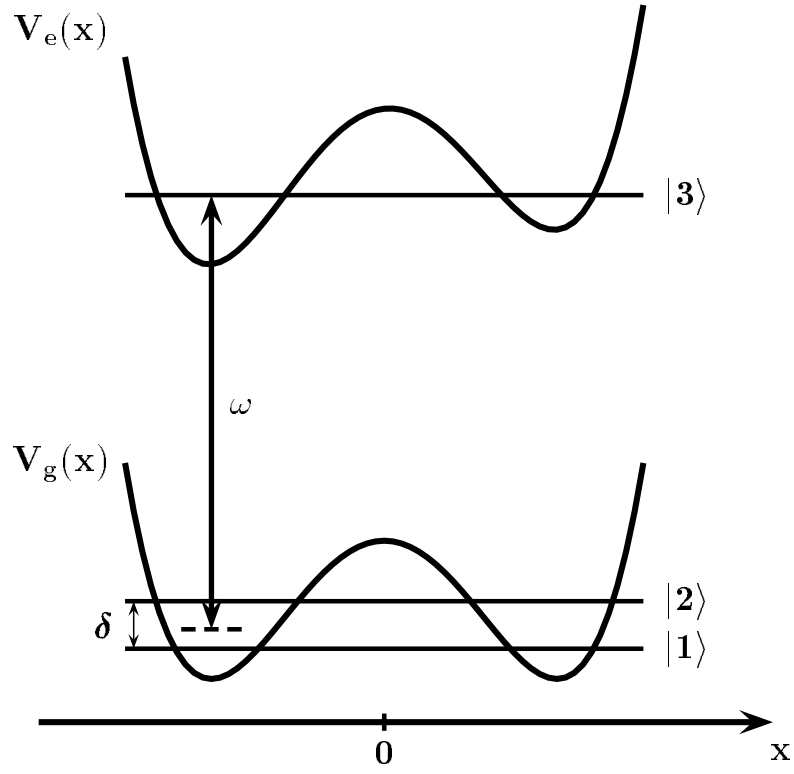


FIG. 1. The quantum system considered here consists of a ground symmetric double-well potential $V_g(x)$ and an excited double-well potential $V_e(x)$. The excited potential is shown to be asymmetric; however, as stated in the text, our results can be applied for more general potentials too. States $|1\rangle$ and $|2\rangle$ are respectively the ground and first excited state of the lower potential and state $|3\rangle$ is a state of the upper potential. These states are coupled by a laser field having angular frequency ω .